Membranes on calibrations

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ABSTRACT: M2-branes can blow up into BPS funnels that end on calibrated intersections of M5-branes. In this quick note, we make the observation that the constraints required for the consistency of these solutions are automatic in Bagger-Lambert-Gustavsson (BLG) theory, thanks to the fundamental identity and the supersymmetry of the calibration. We use this to explain how the previous *ad hoc* fuzzy funnel constructions emerge in this picture, and make some comments about the role of the 3-algebra trace form in the derivation.

Contents

1. Introduction

The difficulty with M-theory is that essentially all the things we know about it are indirect.[1] We know that its low energy limit is 11 dimensional supergravity, we know some of its dual string theories under various compactifications, and we know that its solitonic extended objects are membranes and five-branes. But unlike in the case of string theory D-branes, the lack of a worldsheet description prevents us from explicitly constructing the low energy field theory living on these branes.

But indirect arguments can sometimes go a long way. We can construct (say) membrane solutions in the 11 D supergravity and try to deduce the properties of the M2-worldvolume theory by investigating the SUGRA solution. First of all, since the brane breaks only half of the 32 supersymmetries of 11 D supergravity, we expect that the theory has 16 supersymmetries. Next, we can take a near horizon limit which corresponds to looking at the low energy theory on the worldvolume. The result of the near-horizon limit is an $AdS_4 \times S^7$ background. The isometries of AdS_4 correspond precisely to the conformal group in 2+1 dimensions, and the isometries of S^7 should give rise to an SO(8) R-symmetry. In short, we expect that the theory on the worldvolume of M2-branes is a maximally supersymmetric superconformal theory in 2+1 dimensions with an SO(8) R-symmetry.

But for a while, nobody knew how to build such a theory and in fact it was shown that conventional approaches were doomed to fail [2]. The situation changed recently, when Bagger & Lambert [3] and Gustavsson [4] managed to construct such a theory using ideas inspired by non-associative algebras. This theory is the first ingredient in our work.

The second ingredient consists of BPS funnels constructed using M-branes. Fuzzy funnels are static configurations were one set of branes expands into another. In the case

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[1]See [1] for an exception to this.

of string theory, such solutions have been written down explicitly for the case where D1-branes expand into a single D3-brane (BIon) [5] and into multiple intersecting D3-branes [6]. This latter case can be interpreted also as D1-branes ending on a calibrated cycle wrapped by the D3 [7]. In all these cases the configurations are solutions of the BPS equations (also called Nahm equations) for the low energy theory on the D1-branes which is two dimensional maximally supersymmetric Yang-Mills with $\mathrm{U}(N)$ gauge group.

The generalization of the BIon to the case of the M-theory branes was done first by Basu and Harvey [8]. They wrote down an equation which was an *ad hoc* generalization of the BPS equation, without an underlying supersymmetric action to start with. But it captured the expected features of an M2-brane expanding into an M5. This construction was instrumental in the recent progress in M-brane theory. The Basu-Harvey equation was generalized to the case of multiple intersecting M5s by Berman and Copland in [9]. They found that for general calibrations the configuration has to satisfy some additional constraints (invisible in the Basu-Harvey case) if the BPS equation had to be compatible with the equation of motion.

When the Bagger-Lambert-Gustavsson theory came along, one of its interesting aspects was that it could reproduce the Basu-Harvey equation directly as a BPS equation. The question we address in this paper is how the more general BPS funnels of [9] fit into the theory. In fact, we will see that the analogues of the constraints found in [9] for general calibrations are automatic in the BLG theory because of the structure of the 3-algebra and supersymmetry. Therefore, the previous fuzzy funnel solutions can be constructed in the BLG framework by relating the bracket structures. We also make some comments about the conditions on the trace form of the 3-algebra if we want to derive the BPS equations by following a Bogomol'nyi ansatz on the energy functional. In particular, we discuss the Lorentzian trace forms discussed recently in the literature [18].

Recent work on M2-branes include investigations of their moduli spaces [10], the relation of M2 branes to D2 branes [11], uniqueness theorems of the 3-algebra [12], and explanations for the uniqueness by looking at the boundary of the open membrane version of the theory [13]. Related references are [14]. Useful reviews on the pre-BLG state of the art on M-branes and their interactions are [15].

2. M2-brane field theory

The Lagrangian for Bagger-Lambert-Gustavsson theory is written using the "structure constants" $f^{abc}{}_d$ of a 3-algebra. For the moment, they can be thought of as coefficients that dictate the couplings between the internal components of various fields. With this understanding the Lagrangian is written as

$$\mathcal{L} = -\frac{1}{2}D^\mu X^{aI} D_\mu X_a^I + \frac{i}{2}\bar{\Psi}^a \Gamma^\mu D_\mu \Psi_a + \frac{i}{4}\bar{\Psi}_b \Gamma_{IJ} X_c^I X_d^J \Psi_a f^{abcd}$$
$$-V(X) + \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right). \qquad (2.1)$$

Upper case Latin indices denote the transverse directions of the M2-brane ($I, J, K = 3, \ldots, 10$), and lower case denotes internal indices in the 3-algebra. Greek indices denote

the worldvolume directions ($\mu, \nu, \lambda = 0, 1, 2$). It is useful to think of the Gamma matrices as those of 11 dimensions, constructed as tensor products of SO(2, 1) and SO(8) Gamma matrices. The scalar potential takes the explicit form $V(X) = \frac{1}{12} f^{abcd} f^{efg}{}_d X_a^I X_b^J X_c^K X_e^I X_f^J X_g^K$. The covariant derivative is $D_\mu X_d^I = \partial_\mu X_d^I - \tilde{A}_\mu{}^c{}_d X_c^I$, where $\tilde{A}_\mu{}^c{}_d = A_{\mu ab} f^{abc}{}_d$. The gauge field ensures that the supersymmetry algebra can be closed, but it has no propagating degrees of freedom. This is as it should be because supersymmetry forbids more bosons. On shell, the Lagrangian above is closed under the following SUSY variations

$$\delta X_a^I = i\bar{\epsilon}\Gamma^I \Psi_a$$
$$\delta \Psi_a = D_\mu X_a^I \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X_b^I X_c^J X_d^K f^{bcd}{}_a \Gamma^{IJK} \epsilon \qquad (2.2)$$
$$\delta \tilde{A}_\mu{}^b{}_a = i\bar{\epsilon}\Gamma_\mu \Gamma_I X_c^I \Psi_d f^{cdb}{}_a,$$

if the structure constants satisfy the so-called fundamental identity: $f^{efg}{}_d f^{abc}{}_g = f^{efa}{}_g f^{bcg}{}_d + f^{efb}{}_g f^{cag}{}_d + f^{efc}{}_g f^{abg}{}_d$. In the above, ϵ is the 32 component real spinor of 11 dimensions, which satisfies the condition $\Gamma_{012}\epsilon = \epsilon$ because the other 16 supersymmetries are broken by the M2s. This chirality condition picks out only the $\Gamma_{012}\Psi = -\Psi$ components in the superpartner fermions. The equations of motion that follow from the closure of the SUSY variations (and also from the action) are,

$$\Gamma^\mu D_\mu \Psi_a + \frac{1}{2}\Gamma_{IJ} X_c^I X_d^J \Psi_b f^{cdb}{}_a = 0$$
$$D^2 X_a^I - \frac{i}{2}\bar{\Psi}_c \Gamma^I{}_J X_d^J \Psi_b f^{cdb}{}_a - \frac{\partial V}{\partial X^{Ia}} = 0 \qquad (2.3)$$
$$\tilde{F}_{\mu\nu}{}^b{}_a + \varepsilon_{\mu\nu\lambda}\left(X_c^J D^\lambda X_d^J + \frac{i}{2}\bar{\Psi}_c \Gamma^\lambda \Psi_d \right) f^{cdb}{}_a = 0.$$

with the gauge field strength $\tilde{F}_{\mu\nu}{}^b{}_a = \partial_\nu \tilde{A}_\mu{}^b{}_a - \partial_\mu \tilde{A}_\nu{}^b{}_a - \tilde{A}_\mu{}^b{}_c \tilde{A}_\nu{}^c{}_a + \tilde{A}_\nu{}^b{}_c \tilde{A}_\mu{}^c{}_a$.

When investigating fuzzy funnel solutions we will not be interested in gauge fields and fermions. In this case it is often instructive to write the action in terms of the 3-algebra directly instead of using explicit internal components:

$$\mathcal{L}_B = -\frac{1}{2}\text{Tr}\left(\partial_\mu X^I, \partial^\mu X^I\right) - \frac{1}{12}\text{Tr}([X^I, X^J, X^K], [X^I, X^J, X^K]) \qquad (2.4)$$

This is to be understood in terms of generators T^a ($a = 1, \ldots, N$) such that $X^I \equiv X_a^I T^a$. These generators satisfy $[T^a, T^b, T^c] = f^{abc}{}_d T^d$, and have a trace form $h^{ab} = \text{Tr}\,(T^a, T^b)$. The trace form can be used to raise indices in $f^{abc}{}_d$. In fact to write down the above action (2.1), we have implicitly assumed the existence of such a trace form and assumed also that the f^{abcd} constructed that way is fully antisymmetric in all indices. In terms of the generators, this anti-symmetry condition means that $\text{Tr}(T^a, [T^b, T^c, T^d]) = -\text{Tr}([T^a, T^b, T^c], T^d)$. It should be emphasized that if we forget about the action and merely want to close the supersymmetry variations, we only need the fundamental identity and neither the existence of the trace form nor the full antisymmetry. For future use, we also write down the fundamental identity in terms of the 3-algebra generators:

$$[T^a, T^b, [T^c, T^d, T^e]] = [[T^a, T^b, T^c], T^d, T^e] + [T^c, [T^a, T^b, T^d], T^e] \qquad (2.5)$$
$$+ [T^c, T^d, [T^a, T^b, T^e]],$$

3. Nahm equations and calibrations

We now proceed to construct the BPS equations for fuzzy funnels in Bagger-Lambert-Gustavsson theory following a Bogomol'nyi ansatz. Similar constructions are well known in the literature [9], the point here is only that we would like to work exclusively with the 3-algebra structure.

We are interested in static solutions, so we start with the Hamiltonian for (2.4):

$$E = \frac{1}{2} \int d^2\sigma \left(\mathrm{Tr}(\partial_\sigma X^I, \partial_\sigma X^I) + \frac{1}{3!}\mathrm{Tr}([X^I, X^J, X^K],[X^I, X^J, X^K]) \right) \qquad (3.1)$$

where σ_2 can be gauge-fixed to be X^2. The aim is to write this as

$$E = \frac{1}{2} \int d^2\sigma \left(\mathrm{Tr}\left(\partial_\sigma X^I - \frac{g_{IJKL}}{3!}[X^J, X^K, X^L] \right)^2 + T \right) \qquad (3.2)$$

where T is a total derivative, so that we can identify the perfect square piece with a BPS equation.[2] By direct computation, it turns out that in order to do this we need the constraint

$$\frac{1}{3!} g_{IJKL} g_{IPQR} \mathrm{Tr}([X^J, X^K, X^L],[X^P, X^Q, X^R]) = \mathrm{Tr}([X^I, X^J, X^K],[X^I, X^J, X^K]) \ (3.3)$$

to be satisfied. If this holds, then we can conclude that the BPS configurations of the theory are given by

$$\partial_\sigma X^I - \frac{g_{IJKL}}{3!}[X^J, X^K, X^L] = 0. \qquad (3.4)$$

These equations are the Bagger-Lambert-Gustavsson version of the (generalized) Nahm equations. The coefficients g_{IJKL} are determined by the specific calibration on which the the M2-branes end. Notice that the conventional Nahm equations are written in terms of matrix commutators [6], but here the brackets stand for the 3-product of the 3-algebra.

The coefficients g_{IJKL} characterize the specific BPS solution under consideration. In practice this corresponds to having M-brane configurations that leave some of the supersymmetry unbroken. For BLG theory, from (2.2), setting $\delta\Psi = \partial_\mu X^I \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6}[X^I, X^J, X^K]\Gamma^{IJK}\epsilon$ to zero results in

$$\sum_{I<J<K} [X^I, X^J, X^K]\Gamma^{IJK}(1 - g_{IJKL}\Gamma^{IJKL2})\epsilon = 0, \qquad (3.5)$$

This can be solved as in [6, 9] by defining projectors. We will not repeat the details, the end result is that once we have a set of mutually commuting set of supersymmetries, we can rewrite the above equation in the form

$$\sum_{g_{IJKL}=0} [X^I, X^J, X^K]\Gamma^{IJK}\epsilon = 0. \qquad (3.6)$$

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[2]Note that this assumes implicitly that the trace form in the 3-algebra is positive definite. This means that we are working with the 3-algebra $\mathcal{A}_4$ with structure constants ϵ^{abcd} in this section [12].

The sum is over I, J, K such that $g_{IJKL} = 0$. This can be re-expressed as a set of conditions to be satisfied by the 3-brackets.

Now we will show that the supersymmetry of the calibration, together with the fundamental identity is enough to show that the constraint equations are satisfied in Bagger-Lambert-Gustavsson theory. Together with the identification of the 3-algebra structure with the Nambu 4-bracket proposed in [8, 9], this will imply that the fuzzy funnels on these calibrations are naturally thought of as solutions of BLG. The g_{IJKL} have a direct correspondence to the calibrating forms of the cycle on which the M5 wraps, and therefore are fully antisymmetric in their indices. When $g_{IJKL} = \epsilon_{IJKL}$, we end up with Basu-Harvey and the constraints are identities. For other calibrations g_{IJKL} considered in [9] with more X^I active, things are a bit more complicated. But it turns out that for each of them, after using the conditions arising from supersymmetry (3.6), we can write the constraints (3.3) in terms of the 3-algebra as

$$[[X^L, X^{[I}, X^{J]}], X^{K]}, X^L] = 0, \tag{3.7}$$

$$[[X^M, X^{[I}, X^{J]}], X^K, X^{L]}] = 0. \tag{3.8}$$

Here $[,,]$ stands for antisymmetrization, and the activated I, J, K, L depend crucially on the chosen calibration. But the general structure is all we need to demonstrate our point. The first of these relations follows immediately upon setting $a = e$ (or equivalently, symmetrizing in those indices) in (2.5). After a bit of massaging, the second relation can also be shown to hold due to the fundamental identity. Some useful relations are collected in the appendix.

There is another constraint that we need to check for full consistency of the solution. This arises from the BLG equations of motion (2.3). With the gauge field and the fermion set to zero, the remaining constraint comes from the $A_\mu{}^b{}_a$ equation of motion:

$$X_a^I \partial_\sigma X_b^I f^{abc}{}_d = 0. \tag{3.9}$$

This can be reinterpreted as the statement that the 3-bracket $[X^I, \partial_\sigma X^I, Z]$ vanish for arbitrary Z. If we use the generalized Nahm equation (3.4) to rewrite $\partial_\sigma X^I$, this becomes

$$g_{IJKL}[X^I, [X^J, X^K, X^L], Z] = 0. \tag{3.10}$$

But because of the antisymmetry of g_{IJKL}, this is nothing but the (A.1) form of the fundamental identity.

3.1 Relation to fuzzy 3-spheres

What we have shown is that the BLG theory can in principle have BPS solutions corresponding to various calibrations. Now we show how the 3-algebra structure incorporates the known fuzzy 3-sphere solutions. For a given calibration g_{IJKL}, we get a solution to (3.4) if we can solve the 3-algebra equation

$$\frac{1}{6} g_{IJKL}[A^J, A^K, A^L] = A^I, \tag{3.11}$$

because then $X^I(\sigma) = f(\sigma)A^I$ is a solution to (3.4) for $f(\sigma)$ satisfying $\partial_\sigma f(\sigma) = f^3(\sigma)$. If we imagine that the M5-branes are at $X^2(=\sigma) = 0$, then $f(\sigma) = \frac{i}{\sqrt{2\sigma}}$.

Now we show that the previous solutions [8, 9] can all be seen as specific fuzzy 3-sphere realizations of the 3-algebra equation (3.11) for the case when the 3-algebra is $\mathcal{A}_4$. We first notice from earlier work that the fuzzy 3-sphere coordinates defined in terms of matrices[3] $G^a (a = 1, 2, 3, 4)$ furnish a representation of the 3-algebra $\mathcal{A}_4$ if one takes the 3-bracket to be defined by (upto some constant scalings which we ignore)

$$[G^a, G^b, G^c] \equiv [[G^*, G^a, G^b, G^c]] \sim \epsilon^{abcd} G^d \tag{3.12}$$

Here the 4-bracket operation is the one used by [8, 9] and is defined through ordinary matrix multiplication as the Nambu 4-bracket: $[[A_1, A_2, A_3, A_4]] = \sum_{\text{permutations } \sigma} \text{sgn}(\sigma) A_{\sigma_1} A_{\sigma_2} A_{\sigma_3} A_{\sigma_4}$. Once we have such an explicit representation of $\mathcal{A}_4$, we can use it as a building block, and form various direct sums of these matrices to construct solutions of (3.11) and this is the standard recipe of [9]. Each block gives rise to a single copy of the fuzzy 3-sphere funnel. So the full configuration corresponds to many expanding fuzzy 3-spheres each giving rise to an M5-brane: the fact that the 3-spheres intersect results in the calibrated intersection of M5s.

Before we end this section, we mention that the relevance of the dimensionality of the fuzzy 3-sphere representation here has to be better understood. Naively, if one would translate the BIon intuition for fuzzy 2-spheres to the M2-M5 system, one would expect that the sum of the dimensions of the fuzzy 3-sphere representations in each block should give rise to the total number of M2 branes. But since BLG theory for the 3-algebra $\mathcal{A}_4$ has no free parameters (apart from the level of the Chern-Simons piece) analogous to the rank of the gauge group, this interpretation is far from obvious. In particular, the work of [10, 11] seems to suggest that the theory corresponds to two M2 branes. The fuzzy funnel picture and this picture seem to be at odds. Maybe the mystery can be resolved by a better understanding of the algebraic structures on M2-branes and their representation theory. A reference which tries to address the issue of number of degrees of freedom of fuzzy 3-spheres in this context is [17].

4. Signature of the trace form

The assumption that we made[4] during the Bogomol'nyi construction, namely that the trace is positive definite in the 3-algebra is an incredibly strong one. In fact, it is shown in [12] that the only positive definite finite-dimensional 3-algebra with fully anti-symmetric structure constants of this form is $\mathcal{A}_4$ (and its disconnected copies). Fortunately, as we saw in the last section, this was enough to incorporate all the M2-brane funnel solutions in the literature.

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[3]See [16] for the details of the construction and the definitions of the matrices G^*, G^a.

[4]If we believe the picture that M2-branes have to end on M5-brane configurations, the 3-algebra seems forced to be $\mathcal{A}_4$. One way to see this is to look at the target space of the boundary self-dual string of the BLG theory with boundary. One sees that it is 5+1 dimensional only if the 3-algebra structure constants are those of $\mathcal{A}_4$ [13]. But perhaps the generic non-positive 3-algebras correspond to something more exotic.

It turns out that the constraint relations (3.3) can be obtained without relying on the Bogomol'nyi trick. The BPS equations arise from setting the SUSY variation $\delta\Psi = 0$. Therefore an equation linear in derivatives which preserves some combination of the supersymmetries has to take the form (3.4). One can convince oneself that this is the case by looking at specific configurations of branes (like the ones considered in [9]) that leave specific supersymmetries unbroken and imposing the resulting relations on $\delta\Psi = 0$. Once we accept that the BPS conditions are given by (3.4), one has to check that they are consistent with the equations of motion. From (2.3), we see that the relevant EOM takes the form $\partial^2 X_a^I - \frac{\partial V}{\partial X^{Ia}} = 0$. Taking a derivative of the BPS equation, getting rid of the derivatives, and finally taking a trace, one ends up precisely with the constraint relation (3.3). In this derivation we never needed to use the positivity of the norm.

But of course, the constraints are not the whole story. Lets restrict our attention to *classical, static* solutions. If we demand that a state is BPS, then it means that it satisfies $Q|\psi\rangle = 0$ for some supercharge. This in turn means that $\langle\psi|\{Q,Q\}|\psi\rangle \sim \langle\psi|H|\psi\rangle = 0$ in any positive definite Hilbert space. The energy of any state in a supersymmetric theory is non-negative, but it seems like the Hamiltonian written earlier (3.1) is unbounded below if the trace is not positive definite. So the theory is pathological, at least classically.

Recently, many more solutions to the fundamental identity were constructed with a norm that was Lorentzian [18], by simply augmenting the structure constants of classical Lie algebras. Interestingly, these theories do not suffer from the problem we mentioned above. The scalar part of the Hamiltonian density for these theories can be written as

$$\mathcal{H} = \frac{1}{2}\text{Tr}\left(\partial_\sigma X^I \partial_\sigma X^I\right) - \partial_\sigma X_+^I \partial_\sigma X_-^I$$
$$+ \frac{1}{12}\text{Tr}\left(X_+^I[X^J, X^K] + X_+^J[X^K, X^I] + X_+^K[X^I, X^J]\right)^2 \tag{4.1}$$

where the troublesome negative norm direction has been split off in a "light-cone" notation. The interesting point is that after an integration by parts, X_-^I shows up only as a Lagrange multiplier enforcing the constraint $\partial_\sigma^2 X_+^I = 0$. If we solve this with $X_+^I \sim \sqrt{\lambda}$, where λ is a positive parameter, then the Hamiltonian is schematically that of a $\lambda\phi^4$ theory. Since $\lambda \sim (X_+^I)^2$, we have a positive definite Hamiltonian. It would be interesting to explore these theories along the lines considered here.

Acknowledgments

A. Useful identities

Some ways to rewrite the fundamental identity are collected here,

$$f^{[abc}{}_g f^{e]fg}{}_d = 0, \qquad (A.1)$$

$$f^{abc}{}_g f^{efg}{}_d = 3 f^{ef[a}{}_g f^{bc]g}{}_d. \qquad (A.2)$$

The equivalence of (A.1) to the fundamental identity is shown in Gran, Nilsson and Petersson [11]. Elementary, but useful symmetry relations are

$$[mkl] = \frac{1}{3}(m[kl] + l[mk] + k[lm]) \qquad (A.3)$$

$$[abcde] = \frac{1}{5}([abcd]e + [eabc]d + [deab]c + [cdea]b + [bcde]a), \qquad (A.4)$$

where square brackets stand for anti-symmetrization.

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